A METHOD FOR CORRECTING STAR COUNTS IN MODERATELY OVERCROWDED FIELDS

MODERATELY

M. LÓPEZ–CORREDOIRA, F. GARZÓN, T. MAHONEY
AND P. L. HAMMERSLEY
Instituto de Astrofísica de Canarias
E-38200 La Laguna, Tenerife, Spain

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Abstract. An analytical method of correcting star counts is developed in order to eliminate the effects, to a first order approximation, of star crowding. Also, a comparison with a numerical simulation is performed which agrees with our method.

The finite area of the detector as well as the procedures of source extraction in DENIS limits the minimum distance within which we can separate two stars. This effect can be suppressed, or at least reduced for the most part, when we make a statistical analysis to correct it under some hypothesis of stellar distribution.

We define $A_0(m)$ and $N_0(m) \equiv \int_{-\infty}^{m} dn\ A_0(n)$ as the differential and cumulative star counts inferred directly from the observations without any statistical correction. Also $A(m)$ and $N(m) \equiv \int_{-\infty}^{m} dn\ A(n)$ are the *true* star counts.

A Poissonian distribution for crowded field star distribution is assumed (López-Corredoira *et al.* 1996), where the probability of finding n stars with an aggregate magnitude between m and $m + dm$ in an area S is:

$$dP(n,m) \propto \frac{(S\ A(m)dm)^n}{n!}. \tag{1}$$

Thus, the probability of finding a star with magnitude in the range $(m, m + dm)$ per magnitude bin is

$$P_1(m) \propto S\ A(m), \tag{2}$$

and the probability of finding 2 stars in an area S with respective magnitudes m_1 and m_2 per magnitude bin is

$$P_2(m_1,m_2) \propto \frac{S^2 A(m_1)A(m_2)}{2}. \tag{3}$$

A zero-order estimation is $A(m)dm = A_0(m)dm$ and it is enough for non–crowded fields.

For moderately crowded fields, we must take into account that the possibility of observing two stars instead of one is non-negligible, so a first-order evaluation is obtained according to:

$$\begin{aligned} A(m)dm = {} & A_0(m)dm \times \\ & \times \frac{P_1(m)dm}{P_1(m)dm + \int_{m+dm}^{m_2(m+dm,m)} dm_1\, P_2(m_1, m_2(m_1,m))[m_2(m_1+dm_1,m) - m_2(m_1,m)]} + \\ & + \int_{-\infty}^{m-dm} dm_1\, A_0(m_1) \times \\ & \frac{dm\ P_2(m, m_2(m,m_1))[m_2(m+dm,m_1) - m_2(m,m_1)]}{P_1(m_1)dm_1 + \int_{m_1+dm_1}^{m_2(m_1+dm_1,m_1)} dm_3\, P_2(m_3, m_2(m_3,m_1))[m_2(m_3+dm_3,m_1) - m_2(m_3,m_1)]}, \end{aligned} \tag{4}$$

where $m_2(m_1,m) = -2.5 * \log(10^{-0.4m} - 10^{-0.4m_1})$, and m is the combined magnitude of two stars with magnitudes m_1 and m_2. The probabilities P_1 and P_2 can be calculated from (2) and (3) using first $A = A_0$ and iterating with the result of last expression. The iteration becomes unnecessary since the discrepancy between A and A_0 should not be too high, to preserve a good enough first approximation.

We followed the Bayes theorem for conditioned probability. The first term on the right-hand side of the equality is the amount of single stars observed as one source, which is $A_0(m)dm$ multiplied by the normalized probability of finding only one star associated with this source. It decreases the value of $A(m)dm$ with respect to $A_0(m)dm$. The second term corresponds to stars which are associated with another with magnitude m_2, and are observed as just one source with magnitude m_1, integrated over all possible values of m_1. It increases the star counts.

Further improvements can be performed with approximations up to second or higher order (for more crowded fields), with the same methodology as shown above but by adding the terms integrated over all possibilities of finding three or further stars in S.

Another consideration would be to take the deviation from Poissonian distributions into account. It will be as simple as replacing S by $S \times (1 + \frac{2}{\theta_{\rm min}^2}\int_0^{\theta_{\rm min}} d\theta\theta\omega(\theta))$, where $\omega(\theta)$ is the two–point angular correlation function (see for example Peebles 1980).

An example is performed with expected DENIS counts on a crowded region in the K band up to 14th magnitude (indeed, we need data up to higher magnitudes according to (4), and we use the original $A(m)$ for higher magnitudes. One solution, when we have not any model giving us counts in higher magnitudes, would be correct only up to $\sim$ 12th magnitude or extrapolate $A_0(m)$ for magnitudes between 14 and $\sim$ 16). We take $S = 16$ arcseconds^2 in the K band (approximately in agreement with Holl *et al.* 1994). With a Galaxy model (Wainscoat *et al.* 1992) we estimate $A(m)$ and $A_0(m)$ by means of numerical simulation (magnitude bin $dm = 0.1$). $A_0(m)$ would be the expected DENIS star counts, which after corrections by (4) gives $A_1(m)$. As can be seen in Figs. 1 & 2, $A(m)$ is quite close to $A_1(m)$, *i.e.* we got a considerable improvement on the star counts. In this example we realize that we had an excess of $\sim$ 7% of stars with regard the real distribution up to $m = 13$ (*i.e.* the first term of the correction dominates).

The effect must be stronger in other more crowded regions as well as in I or J (Holl *et al.* 1994), so an application of techniques like this may be suitable when accurate values of star counts are required to compare with the theoretical models.

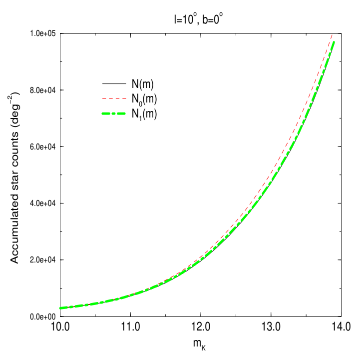

Figure 1. Example of correlation.

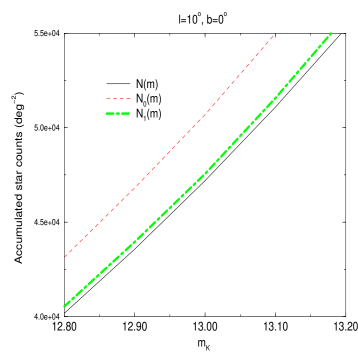

Figure 2. Same example of correction of Fig. 1 with another plot scale.

References

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